Unit VII Review – Discrete Functions – Sequences & Series and Financial Applications June 9/14

1. State whether each of the following sequences is arithmetic, geometric or neither. If arithmetic or geometric, state the common ratio or difference.

a) 5, 18, 57, 174, 525...

neither

b) $\frac{67}{100}, \frac{69}{100}, \frac{71}{100}, \frac{73}{100}, \frac{75}{100}, \frac{77}{100}, \frac{79}{100}, \frac{81}{100}, \frac{83}{100}, \frac{85}{100}$...

arithmetic $d = \frac{1}{50}$

c) $\frac{3}{5}$, 6, 60, 600...

geometric $r = 10$

2. Determine the next three terms in each of the following sequences.

a) 17, 20, 15, 18, 13, 16, 11, ...

+3 -5 +3 -5 +3 -5

$\therefore$ the next three terms are 14, 9, 12

b) $\frac{1}{8}, \frac{1}{27}, \frac{1}{64}, \frac{1}{125}, \frac{1}{216}, \ldots$ $\frac{1}{2^3}, \frac{1}{3^3}, \frac{1}{4^3}, \frac{1}{5^3}, \frac{1}{6^3}, \ldots$

$\therefore$ the next three terms are $\frac{1}{343}, \frac{1}{512}, \frac{1}{729}$

3. Find each of the following terms.

a) the 12th term of the arithmetic sequence 13, 6, -1, -8, -15, ...

$d = -7$

$t_n = a + (n-1)d \Rightarrow$

$t_{12} = 13 + 11(-7)$

$= -64$

4. For each sequence, determine the total number of terms.

a) -18, -2, 14, 30, ..., 126

$d = 16$

$t_n = a + (n-1)d \Rightarrow$

$126 = -18 + (n-1)(16)$

$144 = (n-1)(16)$

$9 = n-1$

$10 = n$

$\therefore$ there are 10 terms.

5. For each sequence, determine the total number of terms.

a) $-8, 4, -2, 1, \ldots, -\frac{1}{128}$

$r = -\frac{1}{2}$

$t_n = a \cdot r^{n-1} \Rightarrow$

$-\frac{1}{128} = -8\left(-\frac{1}{2}\right)^{n-1}$

$\left(-\frac{1}{128}\right)\left(-\frac{1}{8}\right) = \left(-\frac{1}{2}\right)^{n-1}$

$\left(-\frac{1}{2}\right)^7\left(-\frac{1}{2}\right)^3 = \left(-\frac{1}{2}\right)^{n-1}$

$\left(-\frac{1}{2}\right)^{10} = \left(-\frac{1}{2}\right)^{n-1}$

$10 = n-1$

$11 = n$

$\therefore$ there are 11 terms.

6. For each series, calculate:

a) $S_{12} = -5 - 11 - 17 - \ldots$

$a = -5$

$Sn = \frac{n}{2}[2a + (n-1)d] \Rightarrow$ $d = -6$

$S_{12} = \frac{12}{2}\left[2(-5) + (12-1)(-6)\right]$

$= 6[-10 - 66]$

$= 6(-76)$

$= -456$

7. Calculate the sum of the following series.

$$\frac{1}{6}+\frac{1}{18}+\frac{1}{54}+\ldots+\frac{1}{1458}$$

$r=\frac{1}{3}$

$t_n = a \cdot r^{n-1}$ ➡

$$\left[\frac{1}{1458} = \frac{1}{6}\left(\frac{1}{3}\right)^{n-1}\right](\times 6)$$

$$\frac{1}{243} = \left(\frac{1}{3}\right)^{n-1}$$

$$\left(\frac{1}{3}\right)^5 = \left(\frac{1}{3}\right)^{n-1}$$

$n=6$

$S_n = \frac{r \cdot t_n - a}{r-1}$ ⬅

$$S_6 = \frac{\left(\frac{1}{3}\right)\left(\frac{1}{1458}\right)-\frac{1}{6}}{\left(\frac{1}{3}-1\right)}$$

$$= \frac{182}{729}$$

8. *Omit*

9. Alan invests $50 a month at 3%/a compounded monthly. How much will he have in 10 years?

$FV = \frac{R[(1+i)^n - 1]}{i}$ ➡

$i = \frac{0.03}{12} = 0.0025$

$n = 12 \times 10 = 120$

$$= \frac{50[1.0025^{120} - 1]}{0.0025}$$

$= 6987.07$

∴ he will have 6987.07 in 10 years.

10. Meena pays back a $10000 loan with payments every 3 months over 3 years.
If she borrowed the money at 4%/a compounded quarterly, what is her regular payment?

$PV = \frac{R[1-(1+i)^{-n}]}{i}$ ➡

$i = \frac{0.04}{4} = 0.01$

$n = 4 \times 3 = 12$

$$10000 = \frac{R[1-1.01^{-12}]}{0.01}$$

$$\frac{(10000 \times 0.01)}{(1-1.01^{-12})} = R$$

$R = 888.49$

∴ her payments are $888.49.

Pg. 538 # 1 - 6, 8 - 14